

Effect of Aliasing and Throughput at low Sampling rate in Wideband Cognitive Radio Networks

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Abstract—Cognitive Radio (CR) technology is one such technology which has gained popularity in increasing the spectrum access opportunity for secondary users with Wideband spectrum sensing. Taking advantage of the frequency-domain sparsity of the wideband spectrum, a WSS compressed sensing scheme make the spectrum energy efficient but involves the aliasing noise in the signal and also affect the throughput of the signal. In this paper, the problem is formulated in three sections: firstly, sense the spectrum in a multichannel communication environment using relatively low sampling rates, secondly show the effect of throughput on low sampling rate and third is to set a threshold over which the spectrum is energy efficient as well as the aliasing effect is tolerable and probability of missed detection is minimum. Based on simulation and analysis we identified a threshold point where the signal distortion and probability of missed detections are minimum and throughput decreases on increasing sensing time.

Keywords: Cognitive radio network, Wideband spectrum sensing, Digital alias-free signal processing

I. INTRODUCTION

The requirement for higher data rates is increasing as the transition from voice-only communication to multimedia type applications increases. As there are some limitations over the natural frequency spectrum, it is obvious that the present static frequency allocation schemes cannot fulfill the requirements of all the increased higher data rates devices. So new and innovative techniques are required that can

exploit and reuse the available spectrum. *Cognitive radio* is one of those solution to solve spectrum limitation by introducing the new and opportunistic usage of the present frequency bands

that are not heavily occupied by licensed or primary users. One main aspect of cognitive radio is related to autonomously exploiting locally unused spectrum to provide new paths to spectrum access [1]. It is notable that there will be more opportunity for the secondary users (SUs) to access if the greater bandwidth of spectrum is detected. Therefore, wideband spectrum sensing (WSS) scheme [2] is taking attention as a useful and effective approach for detecting multiple bands simultaneously. As we know WSS can be easily carried out in local sensing, that means it can be implemented by any SU independently. Hence, WSS is perfectly suitable with a large number of SUs for large-scale [3].

If comparing the traditional WSS schemes, with the compressed spectrum sensing scheme sampling rate is reduced at the sub-Nyquist rate and sensing time duration is shorter, both the schemes are favorable to energy saving. Although the compressed sensing has been suggested to reduce the sampling rate in WSS, and increase the maximum energy efficiency as we increase the energy efficiency the aliasing effect will also increase and throughput will also be affected. This is the basic idea for this research paper to put a threshold so that the energy efficiency will increase and aliasing effect will also be tolerable and throughput could be maximum.

In this paper, we use a technique that is to sense the activity of the channels in multichannel communication environment based on DASP approach (Digital alias-free signal processing). The spectrum detection approach that will be adopted relies on estimating the spectrum of the incoming signal and sensing its magnitude. The problem related to estimating the spectrum using non-uniformly sampled data has been already studied in few publications [4]. Except [5, 6] there is no paper taken into account with the presence of noise. So, the results examined should be taken with caution when applied to practical situations [7].

The major contributions and requirements of this paper can be summarized as follows:

- The wideband spectrum should be sparse in nature and the received SNR should be greater than the lower limit of SNR.
- We introduce a DASP approach (Digital alias-free signal processing approach) for wideband spectrum sensing in a CR network. Since the compressed sampling is adopted in each sampling channel to wrap the sparse spectrum occupancy map and the effects caused by compressed sampling are analyzed.
- We propose to use different compressive rates in different sampling channels (equivalently different CRs) for improving the spectrum sensing performance and to analyze the effect of noise.
- We examine the effect of throughput as we increasing the sensing time.
 We mathematically analyze the performance of DASP approach and examine the average probabilities of false alarm and detection.

II. WIDEBAND SPECTRUM SENSING SCHEME

A. NETWORK MODE

Our previous paper[3] follows this network mode. These are the basic condition for the spectrum to have and compressed sensing is applied on this network.

B. THROUGHPUT

The principle favorable position of wideband spectrum sensing is its capacity to give better opportunistic throughput RO than meet grave QoS prerequisites for the network secondary users. The opportunistic spectrum gets to operation at a CR includes spectrum sensing taken after by transmitting over the distinguished empty subband(s).

Let $T_{total} = T_{st} + T_{ot}$ be the total access time comprising of the sensing functionality time slot T_{st} and the opportunistic transmission schedule time T_{ot} . It is noticed that, the sensing time T_{st} is expected to join the associated processing time

influenced by the detectors complexity, computational cost and the available processing resources at the SU.

$$T_{st} = \frac{T_{total} - T_{st}}{T_{total}} R_o(1)$$

C. PROBLEM FORMULATION

Assume L be the aggregate number of channels over which information is transmitted in a multichannel communication system. Each channel has a bandwidth of B_c , thus the total data transmission to be checked is $B = LB_c$. All channels central frequencies are known. We expect that the most extreme channel occupancy is low. Our assignment is to deliver a calculation fit for examining the bandwidth B and distinguish which channel(s) are active. The algorithm will apply on sampling rates significantly less than $2B$. Our aim is to use a sampling rate which is smaller than $2B$ but of course to above $2B_A$. Almost Similar task has been already solved in paper [8] by using universal sampling. So that, here in this paper we have searched for algorithms that avoid huge computational costs solutions proposed in [8].

D. SPECTRUM ESTIMATORS

We are considering DASP techniques which are a combination of two steps: signal sampling (non-uniformly) and the second one is calculating its spectrum with the aid of unbiased estimators. The objective of such estimators is given by:

$$Xw(f) = \int_{t_o}^{t_o+T} x(t)w(t)e^{-j2\pi ft} d(t) (2)$$

Here, t_o be the initial time instant of the analyzed signal and T is its width. The windowing function $w(t)$ is normally used to suppress the known Gibbs phenomenon.

In this section, we will review a set of available spectrum estimators and reject the ones that are less appropriate for our purpose.

In paper [8] and [9] they have introduced estimators that are completely based on a total random sampling scheme in which sampling instants are identically distributed, independent random variables whose Probability Distribution Functions (PDF's) includes the whole signal with observation window $[t_o, t_o + T]$. In [9], two estimators were proposed: Weighted Sampled (WS) and Weighted Probability (WP). At the end, the sampling instants PDF's are depends on the used windowing function $w(t)$.

In literature [10] and [11] they introduced estimators that use stratified and antithetical stratified sampling. The two methods present in these papers divides the signal analysis window into sub-intervals under which sample(s) with uniform PDF's are taken. We will refer to those two schemes by jitter sampling thereafter. Therefore, the

remaining candidates are the WS and the two estimators that

III. SPECTRUM ESTIMATORS PERFORMANCE IN PRESENCE OF NOISE

Spectrum estimators which we have studied in [7-10] those all were evaluated in noise-free environments. Although, most of the data transmission systems are related to noise which affects and also limits the performance of the system. Noise is commonly known as zero mean Added White Gaussian Noise (AWGN). Therefore, the transmitted signal which is composed of data $x(t)$ and noise $n(t)$ is represented as $y(t) = x(t) + n(t)$. In this section, we will evaluate the effect of noise on the unbiased nature of the WS estimators and its accuracy also. The given results are related to the estimators that will use jitter (stratified-antithetical) sampling. The sampling used in WS approaches are do not depend on each other and they have identical PDF's which are represented as:

$$P_{ws}(t) = \begin{cases} 1/T & t \in (t_o, t_o + T) \\ 0 & elsewhere \end{cases} \quad (3)$$

WS estimator is defined by:

$$X_{ws}(f) = \frac{T}{N} \sum_{n=1}^N (y(t_n)w(t_n)e^{-j2\pi f t_n}) \quad (4)$$

Where N is the number of the taken samples.

3.1 Unbiased Spectrum Estimator

Here, in this following subsection, we will show that the WS estimator is still unbiased even in the presence of noise. The expected value of the estimators is calculated regards the sample points also the added noise. We noticed that every component of the summation in (3) do not depend on each other and ideally distributed random variables with identical PDF's, hence:

$$E[X_{ws}(f)] = TE[\{x(t) + n(t)\}w(t)e^{-j2\pi f t}] \quad (5)$$

Therefore the estimator is unbiased.

3.2 Accuracy of the Estimator in presence of noise.

According to Chebyshev's inequality, the standard deviation is directly related to the accuracy which states that

$$Pr\{|X - E[X]| \geq \varepsilon\} \leq \frac{\sigma^2}{\varepsilon^2} \quad (6)$$

Here X is a random variable and $\varepsilon > 0$. In this subsection we will calculate the effect of the noise factor on the basis of standard deviations of the WS estimator. The variance is defined as:

$$\sigma^2(f) = E[|X_{ws}(f)|^2] - |X_w(f)|^2 \quad (7)$$

Now $|X_{ws}(f)|^2$

$$= \frac{T^2}{N^2} \sum_{n=1}^N \sum_{m=1}^N y(t_n)y(t_m)w(t_n)w(t_m)e^{-j2\pi f (t_n - t_m)}$$

The expected value of (7) informs the sampling instants and the added noise is evaluated in two stages: when indices are identical i.e. $n = m$ and when they are different, hence:

utilize jitter sampling

$$|X_{ws}(f)|^2 = \frac{T^2}{N^2} \sum_{n=1}^N \sum_{m=1}^N y(t_n)y(t_m)w(t_n)w(t_m)e^{-j2\pi f (t_n - t_m)} + \frac{T^2}{N^2} \sum_{n=1}^N y^2(t_n)w^2(t_n) \quad (8)$$

However, the sampling instants are non-identical and with identical distributions, we can combine (4) and (5) and use it. For $n = m$, expected value of (8) can be written as:

$$\frac{T^2}{N} \{E[x^2(t)w^2(t)] + E[n^2(t)w^2(t)]\} = \frac{T}{N} \{E_{ws} + \sigma^2 E_w\} \quad (9)$$

Where E_{ws} and E_w represent the energy of the windowed signal and the area of the used window respectively. They are defined by:

$$E[x^2(t)w^2(t)] = \frac{1}{N} \int_{t_o}^{t_o+T} [x(t)w(t)]^2 dt = \frac{E_{ws}}{T} \quad (10)$$

$$E[n^2(t)w^2(t)] = \frac{\sigma^2}{N} \int_{t_o}^{t_o+T} w^2(t) dt = \frac{\sigma^2 E_w}{T} \quad (11)$$

When $n \neq m$, the expected value of (8) yields: $\frac{(N-1)}{N} |X_w(f)|^2$. Then

$$E[|X_{ws}(f)|^2] = \frac{T\{E_{ws} + \sigma^2 E_w\}}{N} + \frac{(N-1)}{N} |X_w(f)|^2 \quad (12)$$

By putting equation (11) into (6) we obtain:

$$\sigma^2(f) = \frac{E_{ws} + \sigma^2 E_w}{\alpha} - \frac{|X_w(f)|^2}{N} \quad (13)$$

Where $\alpha = N/T$ is the average sampling rate. Hence,

$$\sigma(f) = \sqrt{\frac{E_{ws} + \sigma^2 E_w}{\alpha} - \frac{|X_w(f)|^2}{N}} \quad (14)$$

$$\sigma_{ws,max} = (E_{ws} + \sigma^2 E_w) / \alpha \quad (15)$$

$\sigma_{ws,max}$ could be describe as a white-noiselike error that is inversely proportional to α . This error is representing the effect or consequences of noise and aliasing on the estimator's accuracy and its relation to the spectrum of the signal $X_w(f)$ is described by Chebyshev's inequality.

IV. PERFORMANCE ANALYSIS AND SIMULATION

In this section, we are analyzing the spectrum that is under sampled and the effect of throughput with the low sampling rate and also sense the spectrum to analyze the effect of aliasing noise with three different conditions of compression rate. Our goal is to sense the spectrum of a multichannel communication system that consists of 10 channels ($L = 10$) that are 2 KHz each ($B_c = 2$ KHz). The monitored range of frequencies i.e. system bandwidth stretches from $f = [500, 1000, 2000]$ Hz.

In figure 1, we consider the compression rate = 0.25, shows the under sampled sinusoidal signal. In figure 2, the

compression rate $\delta = 0.5$ and in figure 3 the compression rate $\delta = 1$. We can see as we are

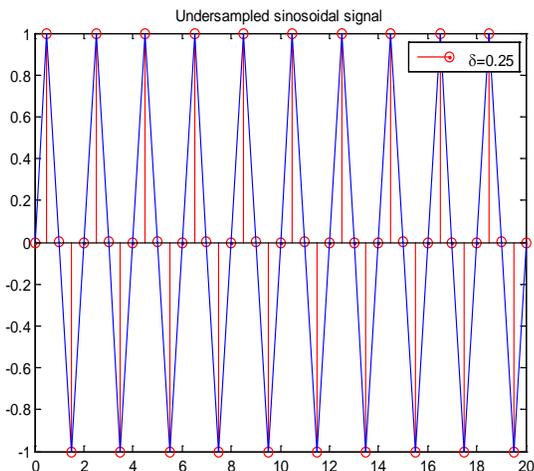


Fig. 1. Under sampled sinusoidal signal at compression rate $\delta=0.25$

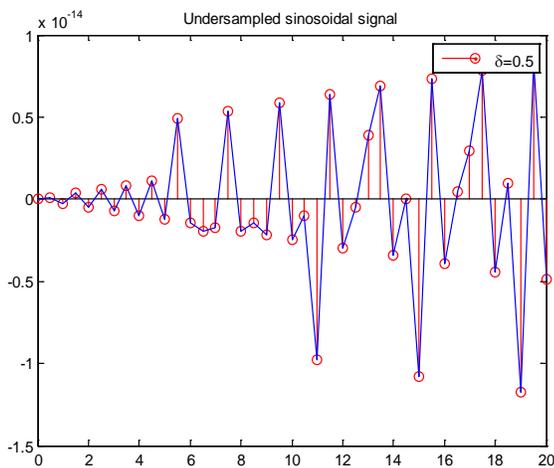


Fig. 2. Under sampled sinusoidal signal at compression rate $\delta=0.5$

increasing the compression rate ($\delta \in 0,1$) the sinusoidal signal gets more distorted because of aliasing effect. In figure 4, on increasing the sensing time the throughput is fallen down. In figure 5, there is combine effect of aliasing noise and AWGN with three different compression rates. The graph is between probability of false alarm and probability of detection which is showing that at minimum value of compression rate that is $\delta=0.25$ the signal is less distorted and because of that the probability of missed detection is almost negligible. As increasing the value of compression rate the signal get more distorted and probability of missed

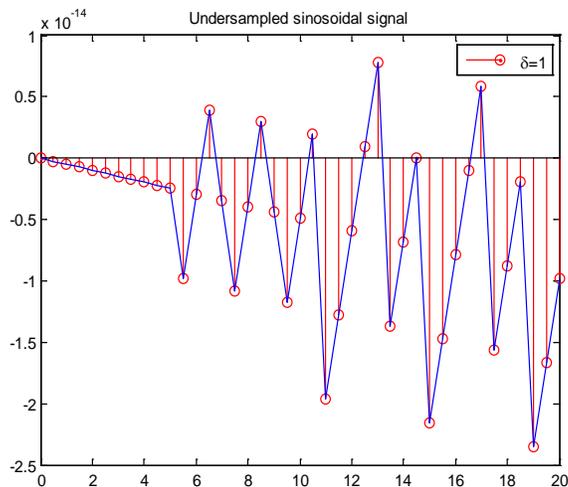


Fig. 3. Under sampled sinusoidal signal at compression rate $\delta=1$

detection is high but after reaching at threshold point, value of missed detection starts decreasing. And that is the basic idea behind this paper is to set a threshold value to avoid the missed detection and correctly sense the spectrum.

V. CONCLUSION

As mentioned earlier in this paper, the increment in compression rate will provide the energy efficient network but also increase the aliasing effect and affect the throughput of the network. In figure 1 to figure 3, we can clearly see the effect of compression rate (as increasing the value of δ signal get more distorted than the previous one). And in figure 4 the graph is between throughputs versus sensing time. Increment in sensing time decreasing the value of throughput (Sensing time will increase when we increases the compression rate means signals get more distorted because of noise). Finally figure 5 explain the effect of noise that is AWGN and aliasing noise which is showing a threshold point whose value is approximately 0.5 after which the probability of missed detection starts decreasing.

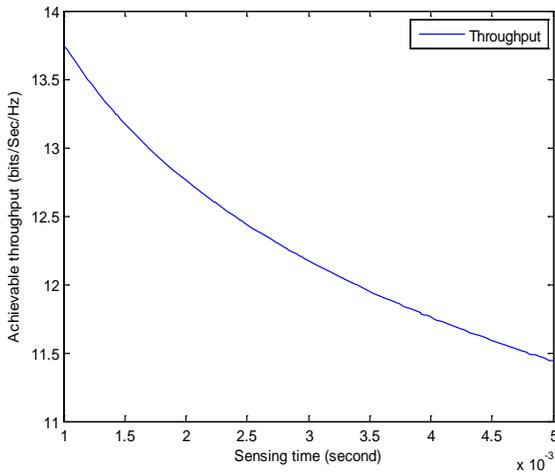


Fig. 4. Throughput versus sensing time

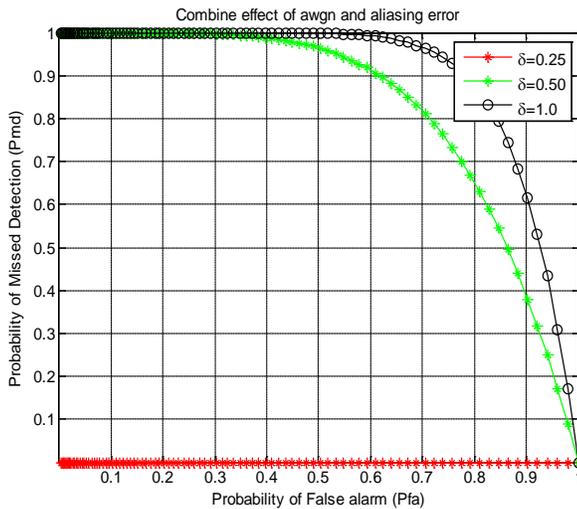


Fig. 5. Combine effect of noise with three different compression rates $\delta = 0.25, 0.5, 1$ showing a threshold point whose value is approximately 0.5 after which the probability of missed detection starts decreasing.

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